TABLE OF CONTENTS

1 ALGEBRA

Table of contents & cheatsheet

1.1. Sequences — 8

Arithmetic: $+/-$ common difference

$$u_n = n^{\text{th}} \text{ term} = u_1 + (n-1)d$$
$$S_n = \text{sum of } n \text{ terms} = \frac{n}{2}\left(2u_1 + (n-1)d\right)$$

with $u_1 = a = 1^{\text{st}}$ term, $d =$ common difference.

Geometric: $\times/\div$ common ratio

$$u_n = n^{\text{th}} \text{ term} = u_1 \cdot r^{n-1}$$
$$S_n = \text{sum of } n \text{ terms} = \frac{u_1(1-r^n)}{(1-r)}$$
$$S_\infty = \text{sum to infinity} = \frac{u_1}{1-r}, \text{ when } -1 < r < 1$$

with $u_1 = a = 1^{\text{st}}$ term, $r =$ common ratio.

Sigma notation
A shorthand to show the sum of a number of terms in a sequence.

Last value of n

$$\sum_{n=1}^{10} 3n-1 \longleftarrow \text{Formula}$$

First value of n

e.g.

$$\sum_{n=1}^{10} 3n-1 = \underbrace{(3\cdot 1)-1}_{n=1} + \underbrace{(3\cdot 2)-1}_{n=2} + \cdots + \underbrace{(3\cdot 10)-1}_{n=10} = 155$$

1.2. Exponents and logarithms — 12

Exponents

$$x^1 = x \qquad x^0 = 1$$
$$x^m \cdot x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$
$$(x^m)^n = x^{m\cdot n} \qquad (x\cdot y)^n = x^n \cdot y^n$$
$$x^{-1} = \frac{1}{x} \qquad x^{-n} = \frac{1}{x^n}$$
$$x^{\frac{1}{2}} = \sqrt{x} \qquad \sqrt{x}\cdot\sqrt{x} = x$$
$$\sqrt{xy} = \sqrt{x}\cdot\sqrt{y} \qquad x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$x^{\frac{m}{n}} = \sqrt[n]{x^m} \qquad x^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{x^m}}$$

Logarithms

$$\log_a a^x = x \qquad a^{\log_a b} = b$$

Let $a^x = b$, isolate x from the exponent: $\log_a a^x = x = \log_a b$

Let $\log_a x = b$, isolate x from the logarithm: $a^{\log_a x} = x = a^b$

Laws of logarithms

I: $\log A + \log B = \log(A \cdot B)$

II: $\log A - \log B = \log\left(\frac{A}{B}\right)$

III: $n \log A = \log(A^n)$

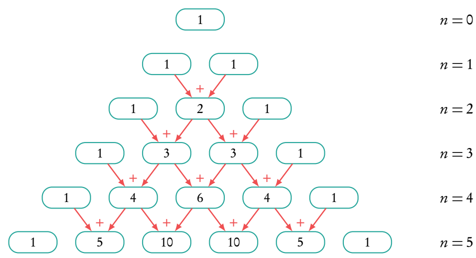
IV: $\log_B A = \frac{\log A}{\log B}$

1.3. Binomial Expansion — 14

In a expansion of a binomial in the form $(a+b)^n$. Each term can be described as $\binom{n}{r} a^n - r b^r$, where $\binom{n}{r}$ is the coefficient.
The full expansion can be written thus

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Find the coefficient using either pascals triangle



Or the nCr function on your calculator

1.1 Sequences

1.1.1 Arithmetic sequence

Arithmetic sequence the next term is the previous number + the common difference (d).

To find the common difference d, subtract two consecutive terms of an arithmetic sequence from the term that follows it, i.e. $u_{(n+1)} - u_n$.

DB 1.1 Use the following equations to calculate the n^{th} term or the sum of n terms.

$$u_n = u_1 + (n-1)d \qquad S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right)$$

with

$$u_1 = a = 1^{\text{st}} \text{ term} \qquad d = \text{common difference}$$

Often the IB requires you to first find the 1^{st} term and/or common difference.

Finding the first term u_1 and the common difference d from other terms.

In an arithmetic sequence $u_{10} = 37$ and $u_{22} = 1$. Find the common difference and the first term.

1.	Put numbers in to n^{th} term formula	$37 = u_1 + 9d$ $1 = u_1 + 21d$
2.	Equate formulas to find d	$21d - 1 = 9d - 37$ $12d = -36$ $d = -3$
3.	Use d to find u_1	$1 - 21 \cdot (-3) = u_1$ u_1

1.1.2 Geometric sequence

Geometric sequence the next term is the previous number multiplied by the common ratio (r).

To find the common ratio, divide any term of an arithmetic sequence by the term that precedes it, i.e. $\dfrac{\text{second term }(u_2)}{\text{first term }(u_1)}$

Use the following equations to calculate the n^{th} term, the sum of n terms or the sum to infinity when $-1 < r < 1$. DB 1.1

$$u_n = n^{\text{th}} \text{ term} = u_1 \cdot r^{n-1} \qquad S_n = \text{sum of } n \text{ terms} = \frac{u_1(1-r^n)}{(1-r)} \qquad S_\infty = \text{sum to infinity} = \frac{u_1}{1-r}$$

again with

$$u_1 = a = 1^{\text{st}} \text{ term} \qquad r = \text{common ratio}$$

Similar to questions on Arithmetic sequences, you are often required to find the 1st term and/or common ratio first.

1.1.3 Sigma notation

Sigma notation is a way to represent the summation of any sequence — this means that it can be used for both arithmetic or geometric series. The notation shows you the formula that generates terms of a sequence and the upper and lower limits of the terms that you want to add up in this sequence.

Last value of n

$$\sum_{n=1}^{10} 3n-1 \longleftarrow \text{Formula}$$

First value of n

e.g. $$\sum_{n=1}^{10} 3n-1 = \underbrace{(3\cdot 1)-1}_{n=1} + \underbrace{(3\cdot 2)-1}_{n=2} + \underbrace{(3\cdot 3)-1}_{n=3} + \cdots + \underbrace{(3\cdot 10)-1}_{n=10} = 155$$

Finding the first term u_1 and common ratio r from other terms.

$\sum_{1}^{5}(\text{Geometric series}) = 3798$, $\sum_{1}^{\infty}(\text{Geometric series}) = 4374$.

Find $\sum_{1}^{7}(\text{Geometric series}) = ?$

1.	Interpret the question	The sum of the first 5 terms of a geometric sequence is 3798 and the sum to infinity is 4374. Find the sum of the first 7 terms
2.	Use formula for sum of n terms	$3798 = u_1 \frac{1-r^5}{1-r}$
3.	Use formula for sum to infinity	$4374 = \frac{u_1}{1-r}$
4.	Rearrange **3.** for u_1	$4374(1-r) = u_1$
5.	Substitute in to **2.**	$3798 = \frac{4374(1-r)\left(1-r^5\right)}{1-r}$
6.	Solve for r	$3798 = 4374\left(1-r^5\right)$ $\frac{3798}{4374} = 1-r^5$ $r^5 = 1-\frac{211}{243}$ $\sqrt[5]{r} = \sqrt[5]{\frac{32}{243}}$ $r = \frac{2}{3}$
7.	Use r to find u_1	$u_1 = 4374\left(1-\frac{2}{3}\right)$ $u_1 = 1458$
8.	Find sum of first 7 terms	$1458\frac{1-\left(\frac{2}{3}\right)^7}{1-\frac{2}{3}} = 4370$