

Review Notes for IB Standard Level Math

Contents

1	Algebra	8
1.1	Rules of Basic Operations	8
1.2	Rules of Roots	8
1.3	Rules of Exponents	8
1.4	Allowed and Disallowed Calculator Functions During the Exam	8
1.5	Sequences and Series	9
1.6	Arithmetic Sequences and Series	9
1.7	Sum of Finite Arithmetic Series $(u_1 + \cdots + u_n)$	9
1.8	Partial Sum of Finite Arithmetic Series $(u_j + \cdots + u_n)$	10
1.9	Geometric Sequences and Series	10
1.10	Sum of Finite Geometric Series	11
1.11	Sum of Infinite Geometric Series	11
1.11.1	Example Involving Sum of Infinite Geometric Series	11
1.12	Sigma Notation	12
1.12.1	Sigma Notation for Arithmetic Series	13
1.12.2	Sigma Notation for Geometric Series	13
1.12.3	Sigma Notation for Infinite Geometric Series	14
1.12.4	Defining Functions Using Sigma Notation	14
1.13	Applications: Compound Interest	15
1.14	Applications: Population Growth	15
1.15	Logarithms	15
1.16	Using Logarithms to Solve Equations	16
1.17	Using Exponentiation to Solve Equations	17
1.18	Logarithm Facts Involving 0 and 1	17
1.19	Laws of Exponents and Logarithms	17
1.20	Change of Base	18
1.21	Powers of Binomials and Pascal's Triangle	18
1.22	Expansion of $(a + b)^n$	19
1.23	The Binomial Theorem	20
1.23.1	Using The Binomial Theorem for a Single Term	21
1.23.2	Example of Using Binomial Theorem	22
1.24	Solving Systems of Three Linear Equations Using Substitution	23
1.25	Solving Systems of Three Linear Equations Using Technology	24
2	Functions and Equations	25
2.1	Sets	25
2.2	Union and Intersection of Sets	25
2.3	Common Sets of Numbers	26
2.4	Intervals of Real Numbers	26
2.5	Concept of Function	27
2.6	Graph of a Function	27
2.7	Domain of a Function	28
2.8	Range of a Function	28
2.9	Composing One Function with Another	29
2.10	Identity Function	30
2.11	Inverse Function	30
2.12	Determining the Inverse Function as Reflection in Line $y = x$	30
2.13	Determining the Inverse Function Analytically	31
2.14	Drawing and Analyzing Graphs with Your Calculator	32
2.14.1	Drawing the Graph of a Function	32

2.14.2	Restricting the Domain of a Graph	32
2.14.3	Zooming Graph to See Exactly What You Want	33
2.14.4	Finding a Maximum Value in an Interval	33
2.14.5	Finding a Minimum Value Value in an Interval	34
2.14.6	Finding the x -Intercepts or “Zeros” of a Graph in an Interval	34
2.14.7	Finding the y -Intercept of a Graph	35
2.14.8	Vertical Asymptotes	35
2.14.9	Graphing Vertical Lines	36
2.14.10	Horizontal Asymptotes	36
2.14.11	Tips to Compute Horizontal Asymptotes of Rational Functions	37
2.14.12	Graphing Horizontal Lines	37
2.14.13	Symmetry: Odd Functions	37
2.14.14	Symmetry: Even Functions	38
2.14.15	Solving Equations Graphically	38
2.15	Transformations of Graphs	40
2.15.1	Horizontal and Vertical Translations	40
2.15.2	Vertical Reflection	41
2.15.3	Horizontal Reflection	42
2.15.4	Vertical Stretch	42
2.15.5	Horizontal Stretch	42
2.15.6	Order Matters When Doing Multiple Transformations in Sequence	43
2.15.7	Graphing the Result of a Sequence of Transformations	44
2.15.8	Determining Point Movement Under a Sequence of Transformations	45
2.15.9	Vector Notation for Function Translation	47
2.16	Quadratic Functions	48
2.16.1	Using the Quadratic Formula to Find Zeros of Quadratic Function	49
2.16.2	Finding the Vertex If You Know the Zeros	49
2.16.3	Graph and Axis of Symmetry	50
2.16.4	Computing the Vertex From the Coefficients	50
2.16.5	Using the Discriminant to Find the Number of Zeros	51
2.16.6	Y-Intercept Form	52
2.16.7	X-Intercept Form	53
2.16.8	Completing the Square to Get Binomial Squared Form	53
2.16.9	Vertex (h, k) Form	54
2.17	Reciprocal Functions	55
2.18	Rational Functions	56
2.19	Exponential Functions	56
2.20	Continuously Compounded Interest	57
2.21	Continuous Growth and Decay	57
2.22	Logarithmic Functions	57
3	Circular Functions and Trigonometry	59
3.1	Understanding Radians	59
3.1.1	Degrees Represent a Part of a Circular Path	59
3.1.2	Computing the Fraction of a Complete Revolution an Angle Represents	59
3.1.3	Attempting to Measure an Angle Using Distance	59
3.1.4	Arc Distance on the Unit Circle Uniquely Identifies an Angle θ	61
3.1.5	Computing the Fraction of a Complete Revolution for Angle in Radians	61
3.2	Converting Between Radians and Degrees	61
3.2.1	Converting from Degrees to Radians	61
3.2.2	Converting from Radians to Degrees	62

3.3	Length of an Arc Subtended by an Angle	62
3.4	Inscribed and Central Angles that Subtend the Same Arc	63
3.5	Area of a Sector	63
3.6	Definition of $\cos \theta$ and $\sin \theta$	64
3.7	Interpreting $\cos \theta$ and $\sin \theta$ on the Unit Circle	65
3.8	Radian Angle Measures Can Be Both Positive and Negative	65
3.9	Remembering the Exact Values of Key Angles on Unit Circle	66
3.10	The Pythagorean Identity	67
3.11	Double Angle Identities	68
3.12	Definition of $\tan \theta$	68
3.13	Using a Right Triangle to Solve Trigonometric Problems	68
3.13.1	Using Right Triangle with an Acute Angle	68
3.13.2	Using Right Triangle with an Obtuse Angle	69
3.14	Using Inverse Trigonometric Functions on Your Calculator	70
3.15	Circular Functions \sin , \cos , and \tan	71
3.15.1	The Graph of $\sin x$	71
3.15.2	The Graph of $\cos x$	71
3.15.3	The Graph of $\tan x$	72
3.15.4	Transformations of Circular Functions	72
3.15.5	Using Transformation to Highlight Additional Identities	73
3.15.6	Determining Period from Minimum and Maximum	74
3.16	Applications of the \sin Function: Tide Example	74
3.17	Applications of the \cos Function: Ferris Wheel Example	76
3.18	Solving Trigonometric Equations in a Finite Interval	78
3.19	Solving Quadratic Equations in \sin , \cos , and \tan	78
3.20	Solutions of Right Triangles	79
3.21	The Cosine Rule	80
3.22	The Sine Rule	80
3.23	Area of a Triangle	81
4	Vectors	82
4.1	Vectors as Displacements in the Plane	82
4.2	Vectors as Displacements in Three Dimensions	82
4.3	Terminology: Tip and Tail	82
4.4	Representation of Vectors	83
4.5	Magnitude of a Vector	83
4.6	Multiplication of a Vector by a Scalar	85
4.7	Negating a Vector	85
4.8	Sum of Vectors	86
4.9	Difference of Vectors	86
4.10	Unit Vectors	88
4.11	Scaling Any Vector to Produce a Parallel Unit Vector	88
4.12	Position Vectors	89
4.13	Determining Whether Vectors are Parallel	89
4.14	Finding Parallel Vector with Certain Fixed Length	90
4.15	Scalar (or “Dot”) Product of Two Vectors	90
4.16	Perpendicular Vectors	90
4.17	Base Vectors for Two Dimensions	90
4.18	Base Vectors for Three Dimensions	91
4.19	The Angle Between Two Vectors	91
4.20	Vector Equation of a Line in Two and Three Dimensions	92

4.21	Vector Equation of Line Passing Through Two Points	93
4.22	Finding the Cartesian Equation from a Vector Line	94
4.23	The Angle Between Two Vector Lines	94
4.24	Distinguishing Between Coincident and Parallel Lines	95
4.25	Finding the Point of Intersection of Two Lines	95
4.25.1	Finding Intersection Between a Line and an Axis	96
5	Statistics	98
5.1	Concepts	98
5.1.1	Population versus Sample	98
5.1.2	Discrete Data versus Continuous Data	98
5.2	Presentation of Data	98
5.2.1	Frequency Distribution Tables	98
5.2.2	Frequency Histograms with Equal Class Intervals	99
5.3	Mean	99
5.4	Median	100
5.5	Mode	101
5.6	Cumulative Frequency	101
5.6.1	Cumulative Frequency Table	101
5.6.2	Cumulative Frequency Graphs	101
5.7	Dispersion	103
5.7.1	Range	103
5.7.2	Quartiles	103
5.7.3	Interquartile Range	103
5.7.4	Box and Whiskers Plots	103
5.7.5	Percentiles	104
5.7.6	Variance	104
5.7.7	Standard Deviation	105
5.8	Use Cumulative Frequency Graph to Find Median and Quartiles	105
5.9	Computing Statistics Using Your Calculator	105
5.9.1	Calculating Statistics for a Single List	106
5.9.2	Calculating Statistics for a List with Frequency	106
5.10	Linear Correlation of Bivariate Data	107
5.10.1	Scatter Diagrams	107
5.10.2	Pearson's Product-Moment Correlation Coefficient r	108
5.10.3	Lines of Best Fit	108
5.10.4	Equation of the Regression Line	108
6	Probability	110
6.1	Probability Concepts	110
6.2	Probability of an Event	110
6.3	Probability of an Event's Not Occurring	110
6.4	Independent, Dependent, and Mutually Exclusive Events	110
6.5	Probability of A and B for Independent Events	111
6.6	Probability of A or B for Mutually Exclusive Events	112
6.7	Probability of Mutually Exclusive Events	112
6.8	Probability of A or B for Non-Mutually Exclusive Events	112
6.9	Lists and Tables of Outcomes	113
6.10	Conditional Probability of Dependent Events	113
6.11	Testing for Independent Events	113
6.12	Venn Diagrams	113
6.13	Tree Diagrams	114

6.14	Probabilities With and Without Replacement	115
6.15	Discrete Random Variables and Their Probability Distributions	115
6.15.1	Explicitly Listed Probabilities	115
6.15.2	Probability Distribution Given by a Function	116
6.15.3	Explicit Probability Distribution Involving an Unknown	116
6.16	Expected Value (Mean), $E(X)$ for Discrete Data	117
6.16.1	Expected Value for a “Fair” Game	118
6.17	Binomial Distribution	119
6.17.1	Using the Calculator for Binomial Distribution Problems	120
6.17.2	Tip for Calculators Whose <code>binomCdf</code> Does Not Have Lower Bound	121
6.18	Mean and Variance of the Binomial Distribution	122
6.18.1	Example of Mean and Variance of Binomial Distribution	122
6.19	Normal Distribution and Curves	123
6.20	Graphing the Normal Distribution and Computing Bounded Area	123
6.21	Standardizing Normal Variables to Get z -values (z -scores)	124
6.22	Using the Calculator for Normal Distribution Problems	125
6.22.1	Tip for Calculators Whose <code>normalCdf</code> Does Not Have Lower Bound	126
6.23	Using Inverse Normal Cumulative Density Function	126
6.24	Determining z Value and σ from the Probability	126
7	Calculus	128
7.1	Overview of Concepts for Differential Calculus	128
7.1.1	Rate of Change in Distance	128
7.1.2	Derivative Gives Instantaneous Rate of Change Using Tangent Lines	130
7.1.3	Relationships Between Function and Derivative	131
7.1.4	Summary of Derivative Concepts	133
7.2	Equations of Tangents and Normals	133
7.3	Notation for Derivatives	134
7.4	Graphing Derivatives with Your Calculator	134
7.4.1	Graphing Derivatives on the TI-Nspire CX	135
7.4.2	Graphing Derivatives on the TI-84 Silver Edition	135
7.5	Computing Derivatives at a Point with Your Calculator	136
7.5.1	Computing Derivative at a Point on the TI-Nspire CX	136
7.5.2	Computing Derivative at a Point on the TI-84 Silver Edition	137
7.6	Rules for Computing Derivatives	137
7.6.1	Derivative of x^n	137
7.6.2	Derivative of $\sin x$	138
7.6.3	Derivative of $\cos x$	138
7.6.4	Derivative of $\tan x$	138
7.6.5	Derivative of e^x	138
7.6.6	Derivative of $\ln x$	138
7.7	Differentiating a Scalar Multiple	138
7.8	Differentiating a Sum or Difference	138
7.9	Example Using Derivation Rules	139
7.10	“Chain Rule”: Differentiating Composed Functions	139
7.11	“Product Rule”: Differentiating Product of Functions	141
7.12	“Quotient Rule”: Differentiating Quotient of Functions	144
7.13	Using the First Derivative to Find Local Maxima and Minima	146
7.14	Analyzing Zeros of Derivative Graph to Find Maxima and Minima	147
7.15	Using the Second Derivative to Determine Concavity and Points of Inflection	148

7.16	Overview of Concepts for Integral Calculus	148
7.16.1	Area Under Curve	149
7.16.2	Relationship Between Derivation and Integration	150
7.17	Anti-Differentiation to Compute Integrals	151
7.17.1	Indefinite Integration as Anti-Differentiation	152
7.18	Rules for Computing the Indefinite Integral (Antiderivative)	152
7.18.1	Indefinite Integral of x^n ($n \in \mathbb{Q}$)	153
7.18.2	Indefinite Integral of $\sin x$	153
7.18.3	Indefinite Integral of $\cos x$	154
7.18.4	Indefinite Integral of $\frac{1}{x}$	154
7.18.5	Indefinite Integral of e^x	154
7.19	Integrating Constant Multiple of a Function	154
7.20	Integrating Sums and Differences of Functions	155
7.21	Integration Using Substitution	155
7.21.1	Simple Example Involving Linear Factor	156
7.21.2	Example Involving Additional Factor of x	157
7.21.3	Example Involving Other Additional Factors	158
7.21.4	Summary of Strategy for Integration by Substitution	158
7.22	Using Additional Information to Determine Constant of Integration	158
7.23	Computing the Definite Integral	160
7.23.1	Computing the Definite Integral Analytically	160
7.23.2	Computing the Definite Integral Using Technology	161
7.23.3	Computing Normal Distribution Probability Using a Definite Integral	162
7.24	Displacement s , Velocity v , and Acceleration a	162
7.25	Determining Position Function from Acceleration	163
7.26	Total Area Under a Curve	164
7.27	Area Between Curves	164
7.27.1	Area Between Curves Using Calculator	164
7.27.2	Area Between Curves Analytically (Without the Calculator)	165
7.28	Net Change in Displacement versus Total Distance Traveled	166
7.28.1	Difference Between Distance and Displacement	166
7.28.2	Computing Total Distance Traveled	166

1 Algebra

1.1 Rules of Basic Operations

You need to be familiar with the rules of basic operations shown in Table 1.

$a(b + c) = ab + ac$	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} = \frac{ad+cb}{bd}$	$-(a + b) = -a - b$
$-(a - b) = b - a$	$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$
$(a + b)(c + d) = ac + ad + bc + bd$	

Table 1: Basic Operations

1.2 Rules of Roots

You need to be familiar with the rules of roots shown in Table 2.

$\sqrt{ab} = \sqrt{a}\sqrt{b}$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$a^{\frac{1}{q}} = \sqrt[q]{a}$	$a^{\frac{p}{q}} = (\sqrt[q]{a})^p$
$a^{-\frac{p}{q}} = \frac{1}{(\sqrt[q]{a})^p}$	$\frac{p}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{p\sqrt{a}}{a}$	$(\sqrt{a})^2 = a$	$(\sqrt[q]{a})^q = a$

Table 2: Rules of Roots

1.3 Rules of Exponents

You need to be familiar with the rules of exponents shown in Table 3.

$a^0 = 1$	$a^1 = a$
$a^b \cdot a^c = a^{b+c}$	$(a^b)^c = a^{b \cdot c}$
$\left(\frac{a}{d}\right)^c = \frac{a^c}{d^c}$	$a^{(-c)} = \frac{1}{a^c}$
$\frac{a^b}{a^c} = a^{(b-c)}$	$\frac{a^b}{d^c} \cdot \frac{d^c}{a^f} = \frac{a^b}{a^f} \cdot \frac{d^c}{d^c} = a^{b-f} \cdot d^{c-e} = \frac{a^{b-f}}{d^{e-c}}$

Table 3: Rules of Exponents

1.4 Allowed and Disallowed Calculator Functions During the Exam

The International Baccalaureate documentation for examiners dated 18/11/2015, provides the following guidance about the functions of the TI NSpire calculator during an exam. The following options in “Press to Test” mode must be ticked and therefore **blocked**: *Limit geometry functions; Disable function and conic grab and move; Disable vector functions including eigenvectors and eigenvalues; Disable isPrime function; Disable differential equation graphing; Disable 3D graphing; Disable implicit graphing, conic templates, conic analysis, and geometric conics.* The following options in “Press to Test” mode must be unticked and therefore **allowed**: *Disable inequality graphing; Limit trigonometric functions; log_b x template and summation functions; Disable Polynomial Root Finder and Simultaneous Equation Solver.*

1.5 Sequences and Series

A *sequence* is an ordered list of numbers. The list can be finite or infinite. We name the sequence with a letter like u , and refer to an individual element (or “term”) of this ordered list using a subscript that represents the ordinal position the particular element occupies in the list. The first term in the sequence u is denoted by u_1 , the second term by u_2 , and so on. The n th term in the sequence is u_n .

You describe a sequence by giving a formula for its n th term. This could be something simple like the sequence each of whose terms is the number 1:

$$u_n = 1$$

More typically, the formula for the n th term involves the variable n in some way. It might be involved in calculating the value of the term like:

$$u_n = n^2 \qquad \text{or} \qquad u_n = \frac{1}{n}$$

The n can also appear in the subscript to reference the values of previous terms. For example, after defining the value of the first and second term, this sequence defines the n th term to be the sum of the previous two terms:

$$\begin{aligned} u_1 &= 1 \\ u_2 &= 1 \\ u_n &= u_{n-1} + u_{n-2} \end{aligned}$$

A *series* is the sum of the terms in a sequence.

1.6 Arithmetic Sequences and Series

A sequence is *arithmetic* if you compute its n th term by adding a constant value d to the previous element:

$$u_n = u_{n-1} + d$$

This value d is known as the *common difference* because if you take any term in the sequence and subtract the value of the preceding one, the difference is d . This is a result of rearranging the previous equation to get:

$$u_n - u_{n-1} = d$$

Given the above definition for u_n involving d , we can also represent the n th element of an arithmetic sequence as the first element to which the common difference d has been added $n - 1$ times.

$$u_n = u_1 + (n - 1)d$$

An *arithmetic series* is the sum of an arithmetic sequence.

1.7 Sum of Finite Arithmetic Series ($u_1 + \cdots + u_n$)

It only makes sense to talk about the sum of a *finite* arithmetic sequence. Since the elements of an *infinite* arithmetic sequence continue growing by the common difference, we don't try to compute the sum of the corresponding infinite arithmetic series because its value would always be ∞ (or $-\infty$ if d is a negative number).

To calculate the sum of the first n terms of a *finite* arithmetic sequence we use the formula:

$$S_n = \frac{n}{2} (u_1 + u_n)$$

This formula comes from observing that the sum of the pair of $u_1 + u_n$ (the first and last terms) is the same as the sum of $u_2 + u_{n-1}$ (the second and penultimate terms), which is the same as the sum of $u_3 + u_{n-2}$ (the third and antepenultimate terms), etc. Since we're pairing up the terms, we end up with $\frac{n}{2}$ pairs having the same sum as $u_1 + u_n$. By multiplying this sum by the number of pairs, we get the sum of all of the terms of the series up to n . Since above we saw that $u_n = u_1 + (n - 1)d$, we can also write this formula as:

$$S_n = \frac{n}{2} (u_1 + (u_1 + (n - 1)d)) = \frac{n}{2} (2u_1 + (n - 1)d)$$

1.8 Partial Sum of Finite Arithmetic Series ($u_j + \dots + u_n$)

Sometimes you might need to calculate the sum of a series starting at a j th term other than the first term, that is where $j > 1$. These kind of problems are easy once you understand that the sum of the first n terms equals the sum of the first $j - 1$ terms plus the sum of the terms from the j th position to the n th. In other words,

$$\begin{aligned} S_n &= (u_1 + \dots + u_{j-1}) + (u_j + \dots + u_n) \\ S_n &= S_{j-1} + S_{j\dots n} \end{aligned}$$

Rearranging the equation we have:

$$S_{j\dots n} = S_n - S_{j-1}$$

Therefore, to find the sum of the series from the j th term through the n th term (inclusive), calculate S_n and subtract from it the value of S_{j-1} .

1.9 Geometric Sequences and Series

A sequence is *geometric* if you compute its n th term by multiplying the previous element by a constant value r :

$$u_n = u_{n-1} \cdot r$$

This value r is known as the *common ratio* because if you take any term in the sequence and divide by the value of the preceding one, the ratio is r . This is a result of rearranging the previous equation to get:

$$\frac{u_n}{u_{n-1}} = r$$

Given the above definition for u_n involving r , we can also represent the n th element of an arithmetic sequence as the first element by which the common ratio r has been multiplied $n - 1$ times.

$$u_n = u_1 \cdot r^{n-1}$$

A *geometric series* is the sum of a geometric sequence.

1.10 Sum of Finite Geometric Series

To calculate the sum of the first n terms of a geometric series, use the formula:

$$S_n = \frac{u_1(r^n - 1)}{r - 1}, \quad r \neq 1$$

1.11 Sum of Infinite Geometric Series

If the absolute value of the common ratio is less than one, then each time we compute the next term of a geometric sequence it is some fraction of the previous term. In other words, the new term is smaller in absolute value than the previous term. Another way to think of this is that the new term is closer to zero than the previous term was. This means we can calculate the sum of the *infinite* geometric series because the terms tend to zero as n gets larger and larger.

To calculate the sum of an infinite geometric series, use the formula:

$$S_\infty = \frac{u_1}{1 - r}, \quad |r| < 1$$

1.11.1 Example Involving Sum of Infinite Geometric Series

As shown in Figure 1, the sides of an outermost square are 16 units. To create a smaller square, connect midpoints of the original square. Consider the sequence A_n of the areas of the shaded triangles in the lower-left corner of each square, formed when the next smaller square is drawn. Find the sum of the areas of these shaded triangles when this process continues infinitely.

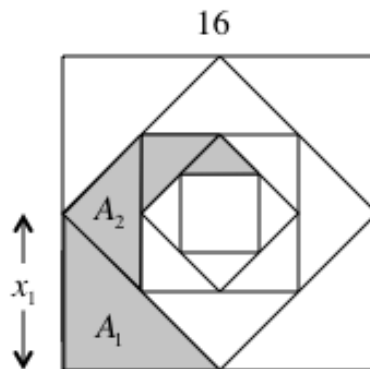


Figure 1: Progressively Smaller Triangle Areas

Since the original side length is 16, the legs of the first shaded triangle are 8, and the area $A_1 = \frac{1}{2} \cdot 8 \cdot 8 = 32$. Using Pythagorean Theorem, we can see that for a square of side-length a , its hypotenuse will always be $\sqrt{a^2 + a^2} = a\sqrt{2}$, so the hypotenuse of the first triangle is $8\sqrt{2}$. This means that the legs of the second triangle are half of this, or $4\sqrt{2}$. Therefore the area of the second triangle is $A_2 = \frac{1}{2} \cdot 4\sqrt{2} \cdot 4\sqrt{2} = 16$. The hypotenuse of the second shaded triangle is $4\sqrt{2}\sqrt{2} = 8$, so the legs of the third triangle are half of this: 4. Hence, the area of the third shaded triangle is $A_3 = \frac{1}{2} \cdot 4 \cdot 4 = 8$. The hypotenuse of the third shaded triangle is $4\sqrt{2}$, so the legs of the fourth shaded triangle are half of this: $2\sqrt{2}$. So, the area of the fourth shaded triangle is $A_4 = \frac{1}{2} \cdot 2\sqrt{2} \cdot 2\sqrt{2} = 4$. So, the pattern that is emerging is that $A = \{32, 16, 8, 4, \dots, 2^{6-n}, \dots\}$.

Considering the sequence A of the triangle areas, we see that each subsequent term is obtained by multiplying the previous term by $\frac{1}{2}$, so the sequence of the areas is a geometric sequence with common ratio $r = \frac{1}{2}$. Since the common ratio is less than one in absolute value, it makes

sense to compute the infinite sum, which is given by the formula in Section 1.11, “Sum of Infinite Geometric Series”:

$$\sum_{n=1}^{\infty} A_n = \frac{A_1}{1-r} = \frac{32}{1-\frac{1}{2}} = 64$$

1.12 Sigma Notation

Sigma notation is another convenient way to represent a series. Instead of saying an English phrase like “the sum of the first 5 terms of the sequence defined by $u_n = 2n^2 + 1$ ”, we can write the same expression using the much more compact formula:

$$\sum_{n=1}^5 2n^2 + 1$$

The \sum symbol is the capital Greek letter sigma, the first letter in the word “sum”. The $n = 1$ below the \sum indicates the variable whose value will be changing to compute the terms in the series, and it also indicates the starting value of this variable. The 5 above the \sum symbol provides the ending value of the variable n specified below the symbol. You compute each term in the sequence by substituting the current value of the changing variable into the expression that follows the \sum symbol. The series is the sum of these terms.

In the example above, the variable is n and its value starts at 1 and goes to 5, incrementing by 1 each time. This means that we can “expand” the compact sigma notation in the example into this expression:

$$\begin{aligned} \sum_{n=1}^5 2n^2 + 1 &= \left(\underbrace{2(1)^2 + 1}_{n=1} \right) + \left(\underbrace{2(2)^2 + 1}_{n=2} \right) + \left(\underbrace{2(3)^2 + 1}_{n=3} \right) + \left(\underbrace{2(4)^2 + 1}_{n=4} \right) + \left(\underbrace{2(5)^2 + 1}_{n=5} \right) \\ &= (2 \cdot 1 + 1) + (2 \cdot 4 + 1) + (2 \cdot 9 + 1) + (2 \cdot 16 + 1) + (2 \cdot 25 + 1) \\ &= (3) + (9) + (19) + (33) + (51) \\ &= 115 \end{aligned}$$

In the example above, by computing the difference between two pairs of consecutive terms, we can observe that the sequence (and hence the series) is not arithmetic because the terms do not share a common difference:

$$\begin{aligned} u_2 - u_1 &= 9 - 3 = 6 \\ u_3 - u_2 &= 19 - 9 = 10 \neq 6 \end{aligned}$$

By computing the ratio between two pairs of consecutive terms, we can also observe that the sequence (and hence the series) is not geometric because the terms do not share a common ratio:

$$\begin{aligned} \frac{u_2}{u_1} &= \frac{9}{3} = 3 \\ \frac{u_3}{u_2} &= \frac{19}{9} \neq 3 \end{aligned}$$

Of course there is nothing special about the letter n and the value of the variable being incremented from the start index to the end index need not start with the value 1. The sigma notation can be used with any letter for the variable and any integer values for the index start



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